Q.2 a. State and explain Gauss Law for electrostatic and derive its differential form.

Answer: 3.2 of Text Book I

b. Define potential and electric flux density. Also derive expression for a point charge in electric field. Point charges 1 mC and -2 mC are located at (3, 2, -1) and (-1, -1, 4), respectively. Calculate the electric force on a 10-nC charge located at (0, 3, 1) and the electric field intensity at that point.

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Answer: 3.1 of Text Book I
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$$\mathbf{F} = \sum_{k=1,2} \frac{QQ_k}{4\pi\varepsilon_0 R^2} \mathbf{a}_R = \sum_{k=1,2} \frac{QQ_k(\mathbf{r} - \mathbf{r}_k)}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_k|^3} \\ = \frac{Q}{4\pi\varepsilon_0} \left\{ \frac{10^{-3}[(0,3,1) - (3,2,-1)]}{|(0,3,1) - (3,2,-1)|^3} - \frac{2.10^{-3}[(0,3,1) - (-1,-1,4)]}{|(0,3,1) - (-1,-1,4)|^3} \right\} \\ = \frac{10^{-3} \cdot 10 \cdot 10^{-9}}{4\pi \cdot \frac{10^{-9}}{36\pi}} \left[\frac{(-3,1,2)}{(9+1+4)^{3/2}} - \frac{2(1,4,-3)}{(1+16+9)^{3/2}} \right] \\ = 9 \cdot 10^{-2} \left[\frac{(-3,1,2)}{14\sqrt{14}} + \frac{(-2,-8,6)}{26\sqrt{26}} \right] \\ \mathbf{F} = -6.507\mathbf{a}_x - 3.817\mathbf{a}_y + 7.506\mathbf{a}_z \, \mathrm{mN}$$

At that point,

$$\mathbf{E} = \frac{\mathbf{F}}{Q}$$

= (-6.507, -3.817, 7.506) $\cdot \frac{10^{-3}}{10 \cdot 10^{-9}}$
$$\mathbf{E} = -650.7\mathbf{a}_x - 381.7\mathbf{a}_y + 750.6\mathbf{a}_z \,\mathrm{kV/m}$$

Q.3 a. Derive the relationship between normal and tangential components at the boundary region in case of perfect dielectrics.

Answer: 6.2 of Text Book I

- b. A wire of diameter 1 mm and conductivity 5×10^7 S/m has 10^{29} free electron s/m³ when an electric field of 10 mV/m is applied. Determine
 - (i) The charge density of free electrons
 - (ii) The current density
 - (iii) The current in the wire
 - (iv) The drift velocity of the electrons. Take the electronic charge as $e = -1.6 \times 10^{-19} C$.

Answer:

(In this particular problem, convection and conduction currents are the same.)

(a)
$$\rho_{\nu} = ne = (10^{29})(-1.6 \times 10^{-19}) = -1.6 \times 10^{10} \text{ C/m}^3$$

(b) $J = \sigma E = (5 \times 10^7)(10 \times 10^{-3}) = 500 \text{ kA/m}^2$
(c) $I = JS = (5 \times 10^5) \left(\frac{\pi d^2}{4}\right) = \frac{5\pi}{4} \cdot 10^{-6} \cdot 10^5 = 0.393 \text{ A}$
(d) Since $J = \rho_{\nu}u$, $u = \frac{J}{\rho_{\nu}} = \frac{5 \times 10^5}{1.6 \times 10^{10}} = 3.125 \times 10^{-5} \text{ m/s}.$

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Q.4 a. Derive the Poisson's and Laplace Equation. Represent Laplace Equation in all three co-ordinate systems.

Answer: 7.1 of Text Book I

Q.5 a. State and explain Ampere's Circuit law.

Answer: 8.2 of Text Book I

b. Explain the concept of Scalar and Vector Magnetic potential.

Answer: 8.6 of Text Book I

Q.6 b. Let us $\mu_1 = 4 \times 10^{-6}$ H/m in Region 1 :- z > 0, $\mu_2 = 7 \times 10^{-6}$ H/m in Region 2 :- z < 0. Let K = $.80\hat{o}_x$ A/m on the surface z = 0 and Field $B_1 = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$ mT in Region 1. Find B_2 in Region 2.

Answer: Page Number 283 of Text Book I

Q.7 a. Express Maxwell's equation in both differential and integral form for a time varying field and explain it.

Answer: 10.3-10.4 of Text Book I

b. The electric field in free space is given by

$$E = 50 \cos (10^8 t + \beta x) \hat{a}_v V/m$$

- (i) Find the direction of wave propagation.
- (ii) Calculate β and the time it takes to travel a distance of ($\lambda/2$).

Answer:

(a) From the positive sign in $(\omega t + \beta x)$, we infer that the wave is propagating along $-\mathbf{a}_x$. This will be confirmed in part (c) of this example.

(b) In tree space, u = c.

$$\beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

or

$\beta = 0.3333$ rad/m

If T is the period of the wave, it takes T seconds to travel a distance λ at speed c. I travel a distance of $\lambda/2$ will take

$$t_1 = \frac{T}{2} = \frac{1}{2} \frac{2\pi}{\omega} = \frac{\pi}{10^8} = 31.42 \text{ ns}$$

Alternatively, because the wave is traveling at the speed of light c,

$$\frac{\lambda}{2} = ct_1$$
 or $t_1 = \frac{\lambda}{2c}$

But

$$\lambda = \frac{2\pi}{\beta} = 6\pi$$

Hence,

$$t_1 = \frac{6\pi}{2(3 \times 10^8)} = 31.42 \text{ ns}$$

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Q.8 With reference to ionosphere and skywave propagation, explain the following terms:

- (i) The virtual height
- (ii) The critical frequency
- (iii) The maximum usable frequency
- (iv) The skip distance

Answer: Page Number 241-242 of Text Book II

TEXTBOOK

1. Engineering Electromagnetics, W. H. Hayt and J. A. Buck, Seventh Edition, Tata McGraw Hill, Special Indian Edition 2006.

2. Electronic Communication Systems, George Kennedy and Bernard Davis, Fourth Edition (1999), Tata McGraw Hill Publishing Company Ltd.